ELECTRON-PION SCATTERING

Abstract

The electron charge is considered to be distributed or extended in space. The differential of the electron charge is set equal to a function of electron charge coordinates multiplied by a four-dimensional differential volume element. The four-dimensional integral of this function is required to equal the electron charge in all Lorentz frames. The pion is treated as having structure and size due to the strong interactions. The result is that for extended electron-structured pion scattering, the S-matrix is the S-matrix for point electron-point pion scattering times an electron form factor and times a pion form factor.

I. INTRODUCTION

Let $x_r'^{\mu} = (x_r'^0, x_r'^1, x_r'^2, x_r'^3)$ denote a spacetime charge point, in the rest frame of an electron charge distribution, and let $x_r^{\mu} = (x_r^0, x_r^1, x_r^2, x_r^3)$ denote the center of the charge distribution. The charge distribution of the electron is assumed to have a well-defined center, which is identified as the argument of the wave function. The shape of the charge

Date: August 20, 2013.

distribution depends on the motion of the charge, and is assumed to be unaffected by any interaction. Sometimes the superscript will be omitted, and we will write $x'_r = (x'^0_r, x'^1_r, x'^2_r, x'^3_r)$ and $x_r = (x^0_r, x^1_r, x^2_r, x^3_r)$. Introduce $\tilde{x}_r = x'_r - x_r$ or equivalently $\tilde{x}^\mu_r = x'^\mu_r - x^\mu_r$. In a frame of reference in which the charge distribution moves with a speed β in the $+x^3$ direction, let $x'_m = (x'^0_m, x'^1_m, x'^2_m, x'^3_m)$ denote a spacetime charge point, and let $x_m = (x^0_m, x^1_m, x^2_m, x^3_m)$ denote the center of the charge distribution. Introduce $\tilde{x}_m = x'_m - x_m$. A Lorentz transformation yields $\tilde{x}^1_r = \tilde{x}^1_m$, $\tilde{x}^2_r = \tilde{x}^2_m$, $\tilde{x}^3_r = \gamma(\tilde{x}^3_m - \beta\tilde{x}^0_m)$, and $\tilde{x}^0_r = \gamma(\tilde{x}^0_m - \beta\tilde{x}^3_m)$ where $\gamma = 1/\sqrt{1-\beta^2}$.

In the rest frame, the electron charge e is equal to $\int \rho_r(\tilde{x}_r)\delta(\tilde{x}^0_r)d^4\tilde{x}_r$ where $\rho_r(\tilde{x}_r)$ is the charge density in the rest frame and δ denotes the delta function.[1] So an element of charge in the rest frame is given by

$$de_r = \rho_r(\tilde{x}_r)\delta(\tilde{x}^0_r)\, d^4\tilde{x}_r. \tag{1}$$

In the m frame, the differential electric charge will be denoted by de_m where de_m is given by $\rho_r(\tilde{x}_m)\delta(\gamma(\tilde{x}^0_m - \beta\tilde{x}^3_m)\, d^4\tilde{x}_m$.[2]

In the next section, the S-matrix for point electron-point pion scattering is calculated. The pion form factor is introduced in the third section, and the S-matrix for extended electron-extended pion is calculated. A short discussion follows.

II. ELECTRON-PION SCATTERING

The calculation of the S-matrix will follow Aitchison and Hey.[3] The probability amplitude for an electron at spacetime point x to exchange a photon of four-momentum q with a pion at spacetime point y is

$$S_{fi} = \int d^4x\, d^4y\, \bar{\phi}_f(x)(-ie\gamma_\mu)\phi_i(x) i D_F(x-y) j^\mu(y) \tag{2}$$

where $-e$ is the electron charge, $\phi_f(x)$ is the final electron wave function, $\gamma_\mu (\mu = 0, 1, 2, 3)$ are the Dirac matrices, $\phi_i(x)$ is the initial electron wave function, and $\bar{\phi}_f = \phi_f^\dagger \gamma^0$. $D_F(x-y)$ is the photon propagator. The pion current $j^\mu(y)$ is

$$j^\mu(y) = ie\big(\phi_F^*(y)\partial_\nu \phi_I(y) - (\partial_\nu \phi_F^*(y))\phi_I(y)\big) \tag{3}$$

where $\phi_F(y)$ is the final pion wave function, and $\phi_I(y)$ is the initial pion wave function. Each wave function is normalized to equal two times its energy in a box of volume V:

$$\phi_i(x) = \sqrt{\frac{1}{V}}\, u_i \exp\left(-ip_i \cdot x\right) \tag{4}$$

where u_i is a four-component spinor, which depends on the initial spin and on the initial four-momentum p_i,

$$\phi_f(x) = \sqrt{\frac{1}{V}}\, u_f \exp\left(-ip_f \cdot x\right) \tag{5}$$

where u_f is a four component spinor, which depends on the final spin and on the final four-momentum p_f,

$$\phi_I(y) = \sqrt{\frac{1}{V}} \exp(-ip_I \cdot y) \tag{6}$$

where p_I is the initial pion four-momentum,

$$\phi_F(y) = \sqrt{\frac{1}{V}} \exp(-ip_F \cdot y) \tag{7}$$

where p_F is the final pion four-momentum. Substitute Eqs. (6), and (7) into Eq. (3), and get

$$j^\mu(y) = \frac{e}{V}(p_F + p_I)^\mu \, \exp\left(i(p_F - p_I) \cdot y\right). \tag{8}$$

The photon propagator is given by

$$D_F(x - y) = \int \frac{\exp\left[-i(x - y) \cdot q\right]}{q^2 + i\epsilon} \frac{d^4q}{(2\pi)^4} \tag{9}$$

Substitute Eqs. (4), (5), (8), and (9) into Eq. (2), and get

$$S_{fi} = \frac{e^2}{V^2}(\bar{u}_f \gamma_\mu u_i)(p_F + p_I)^\mu \int d^4x\, d^4y\, \exp\left(i(p_f - p_i)\cdot x\right)$$
$$\exp\left(-iq\cdot(x-y)\right)\exp\left(i(p_f - p_I)\cdot y\right)\frac{-1}{q^2+i\epsilon}\frac{d^4q}{(2\pi)^4} \quad (10)$$

Integrate over the four components of x and y and get

$$S_{fi} = \frac{e^2}{V^2}(\bar{u}_f \gamma_\mu u_i)(p_F + p_I)^\mu \int (2\pi)^4\delta^4(p_f - p_i - q)$$
$$(2\pi)^4\delta^4(p_F - p_I + q)\frac{-1}{q^2+i\epsilon}\frac{d^4q}{(2\pi)^4} \quad (11)$$

Finally

$$S_{fi} = \frac{e^2}{V^2}(\bar{u}_f \gamma_\mu u_i)(p_I + p_F)^\mu \frac{(2\pi)^4\delta^4(p_F + p_f - p_I - p_i)}{q^2}\,. \qquad (12)$$

where $q = p_f - p_i = p_I - p_F$

III. EXTENDED ELECTRON-STRUCTURED PION SCATTERING

Due to virtual strong interaction effects, the pion will have structure and size. Symmetry arguments suggest that $e(p_F + p_I)^\mu$ should be replaced by $e(p_F+p_I)^\mu F(q^2)$ where $F(q^2)$ is referred to as the pion form factor.[3] Electron size is taken into account by replacing $D_F(x-y)$ by

$D_F(x'-y)$ and replacing the electron charge by the four-dimensional volume integral of de_m. So the S-matrix now is

$$S_{FI} = \int d^4x\, d^4y\, \bar{\phi}_f(x)(-i\, de_m\gamma_\mu)\phi_i(x) iD_F(x'-y)$$
$$(p_F + p_I)^\mu\, eF(q^2)\phi_F^*(y)\phi_I(y). \quad (13)$$

Substitute Eqs. (4), (5), (8), and (9) into Eq. (13), and get

$$S_{FI} = \frac{e^2}{V^2}(\bar{u}_f\gamma_\mu u_i)(p_F + p_I)^\mu \int d^4x\, d^4y\, \exp\left(i(p_f - p_i)\cdot x\right)$$
$$\exp\left(-iq\cdot(x-y)\right)\exp\left(i(p_f - p_I)\cdot y\right)$$
$$\frac{-1}{q^2 + i\epsilon}\frac{d^4q}{(2\pi)^4}F(q^2)\int \exp\left(-iq\cdot\tilde{x}\right)\frac{de_m}{e}. \quad (14)$$

This equation reduces to

$$S_{FI} = S_{fi}F(q^2)F_e(q) \quad (15)$$

where the electron form factor $F_e(q)$ is given by

$$F_e(q) = \int \exp\left(-iq\cdot\tilde{x}\right)\frac{de_m}{e}. \quad (16)$$

By invariance of the form factor

$$F_e(q) = \int \exp\left(-iq_r \cdot \tilde{x}_r\right) \frac{de_r}{e}. \tag{17}$$

where q_r is related to q by a Lorentz transformation.

IV. DISCUSSION

The S-matrix for both the electron and pion with size is the S-matrix for point electron-point pion scattering multiplied by an electron form factor and multiplied by a pion form factor. These form factors should be determined by experiment.

ACKNOWLEDGMENTS

I thank Ben for his many improvements to the paper

References

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[2] http://www.electronformfactor.com/Mott-Rutherford Scattering and Beyond.

[3] I Aitchison and A Hey, *Gauge Theories in Particle Physics* (Adam Hilger Ltd, Bristol, 1982), pp. 62-74.