H-ATOM

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1 INTRODUCTION

For mathematical simplicity, both the proton and electron are chosen to be line charges. The subsequent potential energy is determined, and substituted into the Schrodinger equation. The energy of the 1S level is calculated. Some preliminary calculations are made in the earlier sections.

2 POTENTIAL ENERGY OF THE PROTON AND A POINT ELECTRON

Take the proton charge to be linearly distributed along the z-axis from $z = -a_p$ to $z = +a_p$. The charge per length $\lambda = e/2a_p$. Let $\mathbf{r}$ be the vector from the origin to the point electron, and let r be the corresponding distance. Let $\mathbf{z}$ be the vector from the origin to an element of proton charge, and let z be the corresponding distance. Let $\mathbf{r}'$ be the vector from the element of proton charge to the point electron, and let r' be the corresponding distance. The vectors $\mathbf{r}, \mathbf{z}$, and $\mathbf{r}'$ form a triangle where $r' = \sqrt{r^2 + z^2 - 2rz\cos(\theta)}$, and θ is the angle between $\mathbf{r}$ and $\mathbf{z}$. The differential potential due to an element of proton charge is $d\phi = \lambda\, dz/r'$, so the potential energy of the proton and point electron is

$$V = \frac{-e^2}{2a} \int_{-a_p}^{+a_p} \frac{dz}{\sqrt{r^2 + z^2 - 2rz\cos(\theta)}}. \tag{1}$$

3 SHIFT OF THE 1S ENERGY LEVEL