QUARK EFFECT ON H-ATOM SPECTRUM(1S)

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectrum. There are three quarks (point particles) making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $+e$ is the proton charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$. When only the proton is present, take all three quarks to be at the origin.

For a proton in the hydrogen atom, it will be assumed that the electron attracts the positively charged quarks, and that it repels the negatively charged quark. When the electron is a distance r from the origin, assume the up quark is displaced by a distance δ_+. For the electron closer to the origin, it is expected that δ_+ would increase. In the interest of mathematical simplicity, take δ_+ to be constant. So a positive quark is always assumed to be displaced from the origin on the line from the origin to the electron by a constant distance δ_+ toward the electron. Similarly, the negative quark is always assumed to be displaced from the origin on the extension of the line from the electron through the origin by a constant distance δ_-. The displacements δ_+ and δ_- will be assumed to be much smaller than a_0, the Bohr radius.

Often the Taylor expansion $\exp(-2\delta_\pm/a_0) = 1 - 2\delta_\pm/a_0 + 4\delta_\pm^2/(2a_0^2) + \cdots$ will enter the equations for the energy shifts. Only terms in the energy shifts like $eq_\pm/a_0$ and $eq_\pm\delta_\pm/a_0^2$ will be kept. Terms such as $eq_\pm\delta_\pm^n/a_0^{n+1}$ for $n \geq 2$ are negligible in comparison, and will be discarded.

2 THE POTENTIAL ENERGY OF AN UP QUARK AND THE ELECTRON

Consider the model of section (1) where the up quark has moved a distance δ_+ towards the electron. Let r be the distance from the origin to the electron. Consider the case where $\delta_+ \leq r < \infty$, and define r_+ to be the distance from the quark to the electron. Then $r = r_+ + \delta_+$, and $0 \leq r_+ < \infty$. In the region defined above, the potential energy of the up quark and electron is $V_+ = -eq_+ H(r_+)/r_+$ where the unit step function is defined by $H(x) = 1$ for $x > 0$, and $H(x) = 0$ for $x < 0$..

Consider the case where $0 \leq r \leq \delta_+$, and r_{+-} is the distance from the quark to the electron. Then $r = \delta_+ - r_{+-}$. In this region, the potential energy of the up quark and the electron is $V_+ = -eq_+ H(\delta_+ - r_{+-})/r_{+-}$

The total potential energy of the up quark and the electron is

$$V_+ = -\frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-})\,. \tag{1}$$

3 THE POTENTIAL ENERGY OF A DOWN QUARK AND THE ELECTRON

Let r_- be the distance the quark is from the electron. Then $r_- = r + \delta_-$ where $0 \leq r < \infty$. The total potential energy of the down quark and the electron is

$$V_- = -\frac{eq_-}{r_-} H(r)\,. \tag{2}$$

4 THE HAMILTONIAN

For an up quark at the origin, the unperturbed Hamiltonian of the up quark and the electron is $H_{o+} = p^2/2\mu_+ - eq_+/r$ where μ_+ is the reduced mass of an up quark and the electron. The Hamiltonian of an up quark displaced by the distance δ_+ and an electron is

$$H_+ = \frac{p^2}{2\mu_+} + V_+ = \frac{p^2}{2\mu_+} - \frac{eq_+}{r} + \frac{eq_+}{r} + V_+ = H_{o+} + H'_+ \tag{3}$$

where the perturbing Hamiltonian for an up quark and the electron is

$$H'_+ = +\frac{eq_+}{r} - \frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-}) \tag{4}$$

For the down quark at the origin, the unperturbed Hamiltonian of the down quark and the electron is $H_{o-} = p^2/2\mu_- - eq_-/r$ where μ_- is the reduced mass of the down quark and the electron. The Hamiltonian for the down quark displaced by the distance δ_- and an electron is

$$H_- = \frac{p^2}{2\mu_-} + V_- = \frac{p^2}{2\mu_-} - \frac{eq_-}{r} + \frac{eq_-}{r} + V_- = H_{o-} + H'_- \tag{5}$$

where the perturbing Hamiltonian for the down quark is

$$H'_- = \frac{eq_-}{r} - \frac{eq_-}{r_-} H(r)\,. \tag{6}$$

The perturbing Hamiltonian for the quarks and the electron is $H' = 2H'_+ + H'_-$.

5 ENERGY SHIFT DUE TO H'_+

The energy shift associated with H'_+ is

$$\delta E_+ = \int \psi^*(r) H'_+ \psi(r)\, d^3r = I_{1+} + I_{2+} + I_{3+} \tag{7}$$

where $d^3r = r^2\, \sin(\theta)\, d\theta\, d\phi\, dr$, the unperturbed wave function for the $1S$ energy level is $\psi(r) = 2\exp(-r/a_0)/\sqrt{4\pi a_0^3}$,

$$I_{1+} = \int \frac{4\exp(-2r/a_0)}{a_0^3 4\pi} \Big(\frac{eq_+}{r}\Big) r^2\, \sin(\theta)\, d\theta\, d\phi\, dr = \frac{eq_+}{a_0}\,, \tag{8}$$

$$I_{2+} = -\int \frac{4\exp(-2r/a_0)}{a_0^3} \Big(\frac{eq_+}{r_+} H(r_+)\Big)\, r^2\, dr\,, \text{ and} \tag{9}$$

$$I_{3+} = -\int \frac{4\exp(-2r/a_0)}{a_0^3} \Big(\frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-})\Big)\, r^2\, dr\,. \tag{10}$$

6 CALCULATION OF I_{2+}

Substitute $r = r_+ + \delta_+$ in Eq. (9), and find

$$I_{2+} = -\frac{4eq_+}{a_0^3} \exp(-2\delta_+/a_0) \int_0^\infty \exp(-2r_+/a_0) \Big(r_+ + 2\delta_+ + \frac{\delta_+^2}{r_+}\Big) dr_+\,. \tag{11}$$

I_{2+} is the sum of the following integrals:

$$I_{2+1} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\int_0^\infty r_+\exp(-2r_+/a_0)dr_+ =$$
$$-\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\exp(-2r_+/a_0)\Big(\frac{-r_+a_0}{2}-\frac{a_0^2}{4}\Big)\Big|_0^\infty =$$
$$-4\frac{eq_+}{a_0^3}\Big(1-2\frac{\delta_+}{a_0}+4\frac{\delta_+^2}{2a_0^2}\Big)\frac{a_0^2}{4} = -\frac{eq_+}{a_0}+2\frac{eq_+\delta_+}{a_0^2}\,, \quad (12)$$

$$I_{2+2} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)2\delta_+\int_0^\infty \exp(-2r_+/a_0)dr_+ =$$
$$-\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)2\delta_+\exp(-2r_+/a_0)\Big(\frac{-a_0}{2}\Big)\Big|_0^\infty = -4\frac{eq_+}{a_0^2}\delta_+\,, \text{ and} \quad (13)$$

$$I_{2+3} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\delta_+^2\int_0^\infty \exp(-2r_+/a_0)\frac{dr_+}{r_+}\,. \quad (14)$$

I_{2+3} diverges. The problem is the model introduced in this paper where a quark is considered as a point charge. In a more realistic model, the quarks interact with each other, and this leads to a quark wave function and a quark probability distribution of charge. Think of the hydrogen atom where the potential energy of a point proton and a point electron is put into the Schrodinger equation. The solution is an electron probability distribution. When the point quark is replaced by a probability distribution, the integral converges. The resulting energy contains δ_+^2 which makes this contribution to the energy negligible, so set $I_{2+3}=0$.

Add the results together, and find $I_{2+} = -eq_+/a_0 - 2eq_+\delta_+/a_0^2$.

7 CALCULATION OF I_{3+}

Substitute $r = \delta_+ - r_{+-}$ in Eq. (10). Then

$$I_{3+} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\int_0^{\delta_+}\exp(+2r_{+-}/a_0)\Big(r_{+-}-2\delta_++\frac{\delta_+^2}{r_{+-}}\Big)dr_{+-}\,. \quad (15)$$

I_{3+} is the sum of the following integrals:

$$I_{3+1} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\int_0^{\delta_+} r_{+-}\exp(2r_{+-}/a_0)dr_{+-} =$$
$$-\frac{4eq_+}{a_0^3}\int_0^{\delta_+} r_{+-}dr_{+-} = -\frac{4eq_+}{a_0^3}\frac{\delta_+^2}{2}\,. \quad (16)$$

$$I_{3+2} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)(-2\delta_+)\int_0^{\delta_+}\exp(+2r_{+-}/a_0)dr_{+-} =$$

$$+\frac{4eq_+}{a_0^3}2\delta_+\int_0^{\delta_+} dr_{+-} = 8\frac{eq_+}{a_0^3}\delta_+^2\,. \quad (17)$$

$$I_{3+3} = -\frac{4eq_+}{a_0^3}\exp(-2\delta_+/a_0)\delta_+^2\int_0^{\delta_+}\exp(+2r_{+-}/a_0)\frac{dr_{+-}}{r_{+-}}\,. \quad (18)$$

This integral diverges. For the reasons given at the end of section (6), the integral will converge. All three terms are proportional to δ_+^2, so they will be dropped.

Adding all the contributions to the energy shift from an up quark yields $\delta E_+ = -2eq_+\delta_+/a_0^2\,.$

8 ENERGY SHIFT DUE TO H'_-

The energy shift associated with H'_- is

$$\delta E_- = \int \psi^*(r)H'_-\psi(r)\,d^3r = I_{1-} + I_{2-} \text{ where} \quad (19)$$

$$I_{1-} = \int \frac{4\exp(-2r/a_0)}{a_0^3 4\pi}\Big(\frac{eq_-}{r}\Big)r^2\,\sin(\theta)\,d\theta\,d\phi\,dr = \frac{eq_-}{a_0}\,, \text{ and} \quad (20)$$

$$I_{2-} = -\int_0^\infty \frac{4\exp(-2r/a_0)}{a_0^3}\Big(\frac{eq_-}{r_-}\Big)\,r^2\,dr\,. \quad (21)$$

9 CALCULATION OF I_{2-}

Substitute $r = r_- - \delta_-$ in Eq. (21).Then

$$I_{2-} = -\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)\int_{\delta_-}^\infty \exp(-2r_-/a_0)\Big(r_- - 2\delta_- + \frac{\delta_-^2}{r_-}\Big)dr_-\,. \quad (22)$$

I_{2-} is the sum of the following integrals

$$I_{2-1} = -\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)\int_{\delta_-}^\infty r_-\exp(-2r_-/a_0)dr_- =$$

$$-\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)\exp(-2r_-/a_0)\Big(-\frac{r_-a_0}{2} - \frac{a_0^2}{4}\Big)\Big|_{\delta_-}^\infty = -\frac{eq_-}{a_0} - \frac{2eq_-\delta_-}{a_0^2}\,, \quad (23)$$

$$I_{2-2} = -\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)(-2\delta_-)\int_{\delta_-}^{\infty}\exp(-2r_-/a_0)dr_- =$$

$$-\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)(-2\delta_-)\exp(-2r_-/a_0)\Big(-\frac{a_0}{2}\Big)\Big|_{\delta_-}^{\infty} = +\frac{4eq_-\delta_-}{a_0^2}\ , \text{ and} \tag{24}$$

$$I_{2-3} = -\frac{4eq_-}{a_0^3}\exp(+2\delta_-/a_0)2\delta_-^2\int_{\delta_-}^{\infty}\exp(-2r_-/a_0)\frac{dr_-}{r_-}\ . \tag{25}$$

The integral is convergent, but I_{2-3} contains δ_-^2, so I_{2-3} is negligible and is set equal to 0 .

Adding all the contributions to the energy shift from a down quark yields $\delta E_- = +2eq_-\delta_-/a_0^2$.

10 TOTAL ENERGY SHIFT

The total energy shift of the $1S$ level is the sum of $2\delta E_+$ and δE_-, so

$$\delta E(1S) = -4eq_+\delta_+/a_0^2 + 2eq_-\delta_-/a_0^2 \tag{26}$$

Let m_+ be the mass of an up quark, and let m_- be the mass of the down quark. Take $m_- = 2m_+$(this is approximate). With the center of mass of the quarks at the origin, $2m_+\delta_+ = m_-\delta_-$, so $\delta_- = \delta_+$. Substitute $q_+ = 2e/3$ and $q_- = -e/3$ into Eq. (26), and find

$$\delta E(1S) = -\frac{8e^2\delta_+}{3a_0^2} - \frac{2e^2\delta_+}{3a_0^2} = -\frac{10}{3}\frac{e^2\delta_+}{a_0^2}\ . \tag{27}$$

In future papers, the effect on the hydrogen atom spectrum of both quark displacement as in the model and proton size will be studied.

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