QUARK EFFECT ON H-ATOM SPECTRUM(2P)

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectrum. There are three quarks (point particles) making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $+e$ is the proton charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$. When only the proton is present, take all three quarks to be at the origin.

For a proton in the hydrogen atom, it will be assumed that the electron attracts the positively charged quarks, and that it repels the negatively charged quark. When the electron is a distance r from the origin, assume the up quark is displaced by a distance δ_+. For the electron closer to the origin, it is expected that δ_+ would increase. In the interest of mathematical simplicity, take δ_+ to be constant. So a positive quark is always assumed to be displaced from the origin on the line from the origin to the electron by a constant distance δ_+ toward the electron. Similarly, the negative quark is always assumed to be displaced from the origin on the extension of the line from the electron through the origin by a constant distance δ_-. The displacements δ_+ and δ_- will be assumed to be much smaller than a_0, the Bohr radius.

Often the Taylor expansion $\exp(-\delta_\pm/a_0) = 1 - \delta_\pm/a_0 + \delta_\pm^2/(2a_0^2) + \cdots$ will enter the equations for the energy shifts. Only terms in the energy shifts like $eq_\pm/a_0$ and $eq_\pm\delta_\pm/a_0^2$ will be kept. Terms such as $eq_\pm\delta_\pm^n/a_0^{n+1}$ for $n \geq 2$ are negligible in comparison, and will be discarded.

2 THE POTENTIAL ENERGY OF AN UP QUARK AND THE ELECTRON

Consider the model of section (1) where the up quark has moved a distance δ_+ towards the electron. Let r be the distance from the origin to the electron. Consider the case where $\delta_+ \leq r < \infty$, and define r_+ to be the distance from the quark to the electron. Then $r = r_+ + \delta_+$, and $0 \leq r_+ < \infty$. In the region defined above, the potential energy of the up quark and electron is $V_+ = -eq_+ H(r_+)/r_+$ where the unit step function is defined by $H(x) = 1$ for $x > 0$, and $H(x) = 0$ for $x < 0$..

Consider the case where $0 \leq r \leq \delta_+$. Define r_{+-} to be the distance from the quark to the electron. Then $r = \delta_+ - r_{+-}$. In this region, the potential energy of the up quark and the electron is $V_+ = -eq_+ H(\delta_+ - r_{+-})/r_{+-}$

The total potential energy of the up quark and the electron is

$$V_+ = -\frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-}) \,. \tag{1}$$

3 THE POTENTIAL ENERGY OF A DOWN QUARK AND THE ELECTRON

Let r_- be the distance the quark is from the electron. Then $r_- = r + \delta_-$ where $0 \leq r < \infty$. The total potential energy of the down quark and the electron is

$$V_- = -\frac{eq_-}{r_-} H(r) \,. \tag{2}$$

4 THE HAMILTONIAN

For the proton and quarks at the origin, the unperturbed Hamiltonian of an up quark and the electron is $H_{o+} = p^2/2\mu_+ - eq_+/r$ where μ_+ is the reduced mass of an up quark and the electron. The unperturbed Hamiltonian of the down quark and the electron is $H_{o-} = p^2/2\mu_- - eq_-/r$ where μ_- is the reduced mass of the down quark and the electron.

The Hamiltonian of an up quark displaced by the distance δ_+ and an electron is

$$H_+ = \frac{p^2}{2\mu_+} + V_+ = \frac{p^2}{2\mu_+} - \frac{eq_+}{r} + \frac{eq_+}{r} + V_+ = H_{o+} + H'_+ \tag{3}$$

where the perturbing Hamiltonian for an up quark and the electron is

$$H'_+ = +\frac{eq_+}{r} - \frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-}) \tag{4}$$

The Hamiltonian for the down quark displaced by the distance δ_- and an electron is

$$H_- = \frac{p^2}{2\mu_-} + V_- = \frac{p^2}{2\mu_-} - \frac{eq_-}{r} + \frac{eq_-}{r} + V_- = H_{o-} + H'_- \tag{5}$$

where the perturbing Hamiltonian for the down quark is

$$H'_- = \frac{eq_-}{r} - \frac{eq_-}{r_-} H(r)\,. \tag{6}$$

So the perturbing Hamiltonian for the quarks and the electron is $H' = 2H'_+ + H'_-$.

5 ENERGY SHIFT OF $2P$ LEVEL DUE TO H'_+

The energy shift associated with H'_+ is

$$\delta E_+ = \int \psi^*(r,\theta,\phi) H'_+ \psi(r,\theta,\phi)\, d^3r = I_{1+} + I_{2+} + I_{3+} \tag{7}$$

where $d^3r = r^2\,\sin(\theta)\,d\theta\,d\phi\,dr$, the unperturbed wave function for the $2P$ energy level is $\psi(r,\theta,\phi) = R_{21}(r) Y_{1,m}(\theta,\phi)$,

$$R_{21}(r) = \frac{1}{(2a_0)^{3/2}} \frac{r}{\sqrt{3}a_0} \exp\bigl(-r/(2a_0)\bigr), \tag{8}$$

the $Y_{1,m}(\theta,\phi)$ are normalized spherical harmonics,

$$I_{1+} = \frac{eq_+}{(2a_0)^3} \int_0^\infty \frac{r^2 \exp(-r/a_0)}{3a_0^2\, r} r^2\, dr = \frac{eq_+}{4a_0}\,, \tag{9}$$

$$I_{2+} = -\frac{eq_+}{(2a_0)^3} \int \frac{r^2 \exp(-r/a_0)}{3a_0^2\, r_+} H(r_+)\, r^2\, dr\,, \text{ and} \tag{10}$$

$$I_{3+} = -\frac{eq_+}{(2a_0)^3} \int \frac{r^2 \exp(-r/a_0)}{3a_0^2\, r_{+-}} H(\delta_+ - r_{+-})\, r^2\, dr\,. \tag{11}$$

6 CALCULATION OF I_{2+}

Substitute $r = r_+ + \delta_+$ in Eq. (10), and find

$$I_{2+} = -\frac{eq_+}{(2a_0)^3} \exp(-\delta_+/a_0) \int_0^\infty \frac{\exp(-r_+/a_0)}{3a_0^2} \Bigl(r_+^3 + 4r_+^2\delta_+ + 6r_+\delta_+^2 + 4\delta_+^3 + \frac{\delta_+^4}{r_+}\Bigr) dr_+\,. \tag{12}$$

I_{2+} is the sum of the following integrals:

$$I_{2+1} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_0^\infty r_+^3 \frac{\exp(-r_+/a_0)}{3a_0^2}dr_+ =$$
$$-\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\frac{6a_0^4}{3a_0^2} = -\frac{eq_+}{(2a_0)^3}\Big(1-\frac{\delta_+}{a_0}+\frac{\delta_+^2}{2a_0^2}\Big)\frac{6a_0^2}{3} = -\frac{eq_+}{4a_0}+\frac{eq_+\delta_+}{4a_0^2}\,, \tag{13}$$

$$I_{2+2} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)4\delta_+\int_0^\infty r_+^2 \frac{\exp(-r_+/a_0)}{3a_0^2}dr_+ =$$
$$-\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)4\delta_+\frac{2a_0^3}{3a_0^2} = -\frac{eq_+}{(2a_0)^3}\Big(1-\frac{\delta_+}{a_0}+\frac{\delta_+^2}{2a_0^2}\Big)4\delta_+\frac{2a_0}{3} = -\frac{eq_+\delta_+}{3a_0^2}\,. \tag{14}$$

The next two terms are proportional to higher powers of δ_+, and are negligible, so they will be dropped. The integral in the last term is divergent. The problem is the model introduced in this paper where a quark is considered as a point charge. In a more realistic model, the quarks interact with each other, and this leads to a quark wave function and a quark probability distribution of charge. Think of the hydrogen atom where the potential energy of a point proton and a point electron is put into the Schrodinger equation. The solution is an electron probability distribution. When the point quark is replaced by a probability distribution, the integral converges. The resulting energy contains δ_+^4 which makes this contribution to the energy negligible. Add the results together, and find $I_{2+} = -eq_+/(4a_0) - eq_+\delta_+/(12a_0^2)$.

7 CALCULATION OF I_{3+}

Substitute $r = \delta_+ - r_{+-}$ in Eq. (11), and find

$$I_{3+} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{\delta_+}^0 \frac{\exp(+r_{+-}/a_0)}{3a_0^2}\Big(r_{+-}^3-4r_{+-}^2\delta_+ +6r_{+-}\delta_+^2-4\delta_+^3+\frac{\delta_+^4}{r_{+-}}\Big)(dr_{+-})\,. \tag{15}$$

I_{3+} is the sum of the following integrals: The first is

$$I_{3+1} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_0^{\delta_+} r_{+-}^3 \frac{\exp(r_{+-}/a_0)}{3a_0^2}dr_{+-} =$$
$$-\frac{eq_+}{(2a_0)^3}\int_0^{\delta_+}\frac{r_{+-}^3}{3a_0^2}dr_{+-} = -\frac{eq_+}{(2a_0)^3}\frac{\delta_+^4}{12a_0^2}\,. \tag{16}$$

Since this integral is proportional to δ_+^4, it will be dropped. The next three integrals contain higher powers of δ_+ and will also be dropped. The final integral diverges, but is finite when proton size as described in the previous section is included. The integral is multiplied by δ_+^4, so that term too will be dropped. Finally $I_{3+} = 0$.

Adding all the contributions to the energy shift from an up quark yields $\delta E_+ = -eq_+\delta_+/12a_0^2$.

8 ENERGY SHIFT OF $2P$ LEVEL DUE TO H'_-

The energy shift associated with H_- is

$$\delta E_- = \int \psi^*(r,\theta,\phi) H'_- \psi(r,\theta,\phi)\, d^3r = I_{1-} + I_{2-} \text{ where} \tag{17}$$

$$I_{1-} = \frac{eq_-}{(2a_0)^3} \int_0^\infty \frac{r^3 \exp(-r/a_0)}{3a_0^2}\, dr = \frac{eq_-}{(4a_0)}\,, \text{ and} \tag{18}$$

$$I_{2-} = -\frac{eq_-}{(2a_0)^3} \int_0^\infty \frac{r^4 \exp(-r/a_0)}{3a_0^2 r_-} H(r_- - \delta_-)\, dr \tag{19}$$

9 CALCULATION OF I_{2-}

Substitute $r = r_- - \delta_-$ in Eq, (19), and find

$$I_{2-} = -\frac{eq_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{\delta_-}^\infty \frac{\exp(-r_-/a_0)}{3a_0^2} \Big(r_-^3 - 4r_-^2\delta_- + 6r_-\delta_-^2 - 4\delta_-^3 + \frac{\delta_-^4}{r_-} \Big) dr_-\,. \tag{20}$$

I_{2-} is a sum of the following integrals:

$$I_{2-1} = -\frac{eq_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{\delta_-}^\infty \frac{\exp(-r_-/a_0)}{3a_0^2} r_-^3\, dr_- =$$

$$-\frac{eq_-}{(2a_0)^3} \Big(\frac{\delta_-^3 a_0 + 3\delta_-^2 a_0^2 + 6\delta_- a_0^3 + 6a_0^4}{3a_0^2} \Big) = -\frac{eq_-}{4a_0} - \frac{eq_-\delta_-}{4a_0^2}\,, \tag{21}$$

$$I_{2-2} = -\frac{eq_-}{(2a_0)^3} \exp(+\delta_-/a_0)(-4\delta_-) \int_{\delta_-}^\infty \frac{\exp(-r_-/a_0)}{3a_0^2} r_-^2\, dr_- =$$

$$\frac{eq_-}{(2a_0)^3} 4\delta_- \Big(\frac{\delta_-^2 a_0 + 2\delta_- a_0^2 + 2a_0^3}{3a_0^2} \Big) = +\frac{eq_-\delta_-}{3a_0^2}\,. \tag{22}$$

The remaining integrals contain high powers of δ_-, so they will be dropped. Add the results together, and find $I_{2-} = -eq_-/(4a_0) + eq_-\delta_-/(12a_0^2)$. Then the contribution to the energy shift from the down quark is $\delta E_- = eq_-\delta_-/12a_0^2$.

10 TOTAL ENERGY SHIFT

The total energy shift of the $2P$ level is the sum of $2\delta E_+$ and δE_-, so

$$\delta E(2P) = -2eq_+\delta_+/12a_0^2 + eq_-\delta_-/12a_0^2 \tag{23}$$

Let m_+ be the mass of an up quark, and let m_- be the mass of the down quark. Take $m_- = 2m_+$(this is approximate). With the center of mass of the quarks at the origin, $2m_+\delta_+ = m_-\delta_-$, so $\delta_- = \delta_+$. Substitute $q_+ = 2e/3$ and $q_- = -e/3$ into Eq. (23), and find

$$\delta E(2P) = -\frac{4e^2\delta_+}{36a_0^2} - \frac{e^2\delta_+}{36a_0^2} = -\frac{5}{36}\frac{e^2\delta_+}{a_0^2}\,. \tag{24}$$

In future papers, the effect on the hydrogen atom spectrum of both quark displacement as in the model and proton size will be studied.

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