QUARK EFFECT ON H-ATOM SPECTRA(2S)

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectrum. There are three quarks (point particles) making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $+e$ is the proton charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$. When only the proton is present, take all three quarks to be at the origin.

For a proton in the hydrogen atom, it will be assumed that the electron attracts the positively charged quarks, and that it repels the negatively charged quark. When the electron is a distance r from the origin, assume the up quark is displaced by a distance δ_+. For the electron closer to the origin, it is expected that δ_+ would increase. In the interest of mathematical simplicity, take δ_+ to be constant. So a positive quark is always assumed to be displaced from the origin on the line from the origin to the electron by a constant distance δ_+ toward the electron. Similarly, the negative quark is always assumed to be displaced from the origin on the extension of the line from the electron through the origin by a constant distance δ_-. The displacements δ_+ and δ_- will be assumed to be much smaller than a_0, the Bohr radius.

Often the Taylor expansion $\exp(-\delta_\pm/a_0) = 1 - \delta_\pm/a_0 + \delta_\pm^2/(2a_0^2) + \cdots$ will enter the equations for the energy shifts. Only terms in the energy shifts like $eq_\pm/a_0$ and $eq_\pm\delta_\pm/a_0^2$ will be kept. Terms such as $eq_\pm\delta_\pm^n/a_0^{n+1}$ for $n \geq 2$ are negligible in comparison, and will be discarded.

2 THE POTENTIAL ENERGY OF AN UP QUARK AND THE ELECTRON

Consider the model of section (1) where the up quark has moved a distance δ_+ towards the electron. Let r be the distance from the origin to the electron. Consider the case where $\delta_+ \leq r < \infty$, and define r_+ to be the distance from the quark to the electron. Then $r = r_+ + \delta_+$, and $0 \leq r_+ < \infty$. In the region defined above, the potential energy of the up quark and electron is $V_+ = -eq_+ H(r_+)/r_+$ where the unit step function is defined by $H(x) = 1$ for $x > 0$, and $H(x) = 0$ for $x < 0$..

Consider the case where $0 \leq r \leq \delta_+$, and r_{+-} is the distance from the quark to the electron. Then $r = \delta_+ - r_{+-}$. In this region, the potential energy of the up quark and the electron is $V_+ = -eq_+ H(\delta_+ - r_{+-})/r_{+-}$

The total potential energy of the up quark and the electron is

$$V_+ = -\frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-})\,. \tag{1}$$

3 THE POTENTIAL ENERGY OF A DOWN QUARK AND THE ELECTRON

Let r_- be the distance the quark is from the electron. Then $r_- = r + \delta_-$ where $0 \leq r < \infty$. The total potential energy of the down quark and the electron is

$$V_- = -\frac{eq_-}{r_-} H(r)\,. \tag{2}$$

4 THE HAMILTONIAN

For an up quark at the origin, the unperturbed Hamiltonian of the up quark and the electron is $H_{o+} = p^2/2\mu_+ - eq_+/r$ where μ_+ is the reduced mass of an up quark and the electron. The Hamiltonian of an up quark displaced by the distance δ_+ and an electron is

$$H_+ = \frac{p^2}{2\mu_+} + V_+ = \frac{p^2}{2\mu_+} - \frac{eq_+}{r} + \frac{eq_+}{r} + V_+ = H_{o+} + H'_+ \tag{3}$$

where the perturbing Hamiltonian for an up quark and the electron is

$$H'_+ = +\frac{eq_+}{r} - \frac{eq_+}{r_+} H(r_+) - \frac{eq_+}{r_{+-}} H(\delta_+ - r_{+-}) \tag{4}$$

For the down quark at the origin, the unperturbed Hamiltonian of the down quark and the electron is $H_{o-} = p^2/2\mu_- - eq_-/r$ where μ_- is the reduced mass of the down quark and the electron. The Hamiltonian for the down quark displaced by the distance δ_- and an electron is

$$H_- = \frac{p^2}{2\mu_-} + V_- = \frac{p^2}{2\mu_-} - \frac{eq_-}{r} + \frac{eq_-}{r} + V_- = H_{o-} + H'_- \tag{5}$$

where the perturbing Hamiltonian for the down quark and the electron is

$$H'_- = +\frac{eq_-}{r} - \frac{eq_-}{r_-} H(r)\,. \tag{6}$$

5 ENERGY SHIFT DUE TO H'_+

The energy shift associated with H'_+ is

$$\delta E_+ = \int \psi^*(r) H'_+ \psi(r)\, d^3r = I_{1+} + I_{2+} + I_{3+} \tag{7}$$

where $d^3r = r^2 \sin(\theta)\, d\theta\, d\phi\, dr$, the unperturbed wave function for the $2S$ energy level is $\psi_{(2,0,0)}(r) = (2 - r/a_0)\exp(-r/2a_0)/(\sqrt{4\pi(2a_0)^3})$,

$$I_{1+} = \int_0^\infty \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) \frac{\exp(-r/a_0)}{(2a_0)^3 4\pi} \left(\frac{eq_+}{r}\right) r^2 \sin(\theta)\, d\theta\, d\phi\, dr\,, \tag{8}$$

$$I_{2+} = \int \left(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2}\right) \frac{\exp(-r/a_0)}{(2a_0)^3} \left(\frac{-eq_+ H(r_+)}{r_+}\right) dr, \text{ and} \tag{9}$$

$$I_{3+} = \int \left(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2}\right) \frac{\exp(-r/a_0)}{(2a_0)^3} \left(\frac{-eq_+ H(\delta_+ - r_{+-})}{r_{+-}}\right) dr_{+-} \tag{10}$$

6 CALCULATION OF I_{1+}

I_{1+} is the sum of the following integrals:

$$I_{1+1} = \frac{eq_+}{8a_0^3} \int_0^\infty 4r \exp(-r/a_0)\, dr = \frac{eq_+}{2a_0^3}\,, \tag{11}$$

$$I_{1+2} = -\frac{eq_+}{8a_0^4} \int_0^\infty 4r^2 \exp(-r/a_0)\, dr = -\frac{eq_+}{a_0^3}\,, \text{and} \tag{12}$$

$$I_{1+3} = -\frac{eq_+}{8a_0^5} \int_0^\infty r^3 \exp(-r/a_0)\, dr = +\frac{3eq_+}{4a_0^3}\,. \tag{13}$$

Add the results, and find $I_{1+} = eq_+/4a_0$.

7 CALCULATION OF I_{2+}

Substitute $r = r_+ + \delta_+$ in Eq. (9). Then I_{2+} is the sum of the following integrals:

$$I_{2+1} = -\frac{eq_+}{8a_0^3}\exp(-\delta_+/a_0)\int_0^\infty 4\Big(r_+ + 2\delta_+ + \frac{\delta_+^2}{r_+}\Big)\exp(-r_+/a_0)dr_+\,, \quad (14)$$

$$I_{2+2} = +\frac{eq_+}{8a_0^4}\exp(-\delta_+/a_0)\int_0^\infty 4\Big(r_+^2 + 3r_+\delta_+ + 3\delta_+^2 + \frac{\delta_+^3}{r_+}\Big)\exp(-r_+/a_0)dr_+, \text{and}\,,\ . \quad (15)$$

$$I_{2+3} = -\frac{eq_+}{8a_0^4}\exp(-\delta_+/a_0)\int_0^\infty \Big(r_+^3 + 4r_+^2\delta_+ + 6r_+\delta_+^2 + 4\delta_+^3 + \frac{\delta_+^3}{r_+}\Big)\exp(-r_+/a_0)dr_+, \quad (16)$$

Integrals with r_+ in the denominator are divergent. The problem is the model introduced in this paper where a quark is considered as a point charge. In a more realistic model, the quarks interact with each other, and this leads to a quark wave function and a quark probability distribution of charge. Think of the hydrogen atom where the potential energy of a point proton and a point electron is put into the Schrodinger equation. The solution is an electron probability distribution. When the point quark is replaced by a probability distribution, the integral converges. All integrals multiplied by δ_+^n where $n \geq 2$ will be discarded since $\delta_+ << a_0$.

I_{2+1} is the sum of the following integrals:

$$I_{2+11} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_0^\infty 4r_+\exp(-r_+\,dr_+ = \frac{eq_+}{2a_0^3}\Big(1 - \frac{\delta_+}{a_0}\Big)a_0^2 = -\frac{eq_+}{2a_0} + \frac{eq_+\delta_+}{2a_0^2}, \text{ and} \quad (17)$$

$$I_{2+12} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_0^\infty 8\delta_+\exp(-r_+\,dr_+ = \frac{eq_+}{a_0^3}\Big(1 - \frac{\delta_+}{a_0}\Big)\delta_+a_0 = -\frac{eq_+\delta_+}{a_0^2} \quad (18)$$

So $I_{2+1} = -eq_+/(2a_0) - eq_+\delta_+/(2a_0^2)$.

I_{2+2} is the sum of the following integrals:

$$I_{2+21} = +\frac{eq_+}{(2a_0)^4}\exp(-\delta_+/a_0)\int_0^\infty 4r_+^2\exp(-r_+/a_0)dr_+ = +\frac{eq_+}{2a_0^4}\Big(1 - \frac{\delta_+}{a_0}\Big)2a_0^3 = +\frac{eq_+}{a_0} - \frac{eq_+\delta_+}{a_0^2} \quad (19)$$

$$I_{2+22} = +\frac{eq_+}{(2a_0)^4}\exp(-\delta_+/a_0)\int_0^\infty 12\delta_+ r_+ \exp(-r_+/a_0)dr_+ = +\frac{3eq_+\delta_+}{2a_0^2} \tag{20}$$

Add the results together, and find $I_{2+2} = +eq_+/(a_0) + eq_+\delta_+/(2a_0^2)$.

I_{2+3} is the sum of the following integrals:

$$I_{2+31} = -\frac{eq_+}{8a_0^4}\exp(-\delta_+/a_0)\int_0^\infty r_+^3 \exp(-r_+/a_0)dr_+ = \\ -\frac{eq_+}{8a_0^4}\Big(1-\frac{\delta_+}{a_0}\Big)6a_0^4 = -\frac{3eq_+}{4a_0} + \frac{3eq_+\delta_+}{4a_0^2}, \text{ and} \tag{21}$$

$$I_{2+32} = -\frac{eq_+}{8a_0^4}\exp(-\delta_+/a_0)\int_0^\infty 4r_+^2\delta_+ \exp(-r_+/a_0)dr_+ = \\ -\frac{4eq_+\delta_+}{8a_0^4}\Big(1-\frac{\delta_+}{a_0}\Big)2a_0^3 = -\frac{eq_+\delta_+}{a_0^2}\,. \tag{22}$$

Add the results , and find $I_{2+3} = -3eq_+/(4a_0) - eq_+\delta_+/(4a_0^2)$.

Finally $I_{2+} = I_{2+1} + I_{2+2} + I_{2+3} = -eq_+/(4a_0) - eq_+\delta_+/(4a_0^2)$.

8 CALCULATION OF I_{3+}

Substitute $r = \delta_+ - r_{+-}$ in Eq. (10). Then

$$I_{3+} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_0^{\delta_+}\Big(4r_{+-} - 8\delta_+ - \frac{(4r_{+-}^2 - 12r_{+-}\delta_+)}{a_0} + \frac{r_{+-}^3 - 4r_{+-}^2\delta_+}{a_0^2}\Big) \\ \exp(+r_{+-}/a_0)dr_{+-}\,. \tag{23}$$

For $0 \le r_{+-} \le \delta_+$, $\exp(-r_{+-}/a_0) \approx 1$. So all terms are proportional to δ_+^2 and higher powers, so I_{3+} is negligible, and will be dropped.

All the contributions to the energy shift from an up quark and the electron yields $\delta E_+ = -eq_+\delta_+/(4a_0^2)$.

9 ENERGY SHIFT DUE TO H'_-

The energy shift associated with H'_- is

$$\delta E_- = \int \psi^*(r)H'_-\psi(r)\, d^3r = I_{1-} + I_{2-} \text{ where} \tag{24}$$

$$I_{1-} = \int \Big(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\Big) \frac{\exp(-r/a_0)}{(2a_0)^3 4\pi} \Big(\frac{eq_-}{r}\Big) r^2 \sin(\theta)\, d\theta\, d\phi\, dr = \frac{eq_-}{4a_0}, \text{ and} \quad (25)$$

$$I_{2-} = -\int \Big(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2}\Big) \frac{\exp(-r/a_0)}{(2a_0)^3} \Big(\frac{eq_- H(r)}{r_-}\Big) dr\,. \quad (26)$$

10 CALCULATION OF I_{2-}

Substitute $r = r_- - \delta_-$ in Eq.(26). Then I_{2-} is the sum of the following integrals:

$$I_{2-1} = -\frac{eq_-}{8a_0^3} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 4\Big(r_- - 2\delta_- + \frac{\delta_-^2}{r_-}\Big) \exp(-r_-/a_0)\, dr_-\,, \quad (27)$$

$$I_{2-2} = +\frac{eq_-}{8a_0^4} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 4\Big(r_-^2 - 3r_-\delta_- + 3\delta_-^2 - \frac{\delta_-^3}{r_-}\Big) \exp(-r_-/a_0)\, dr_-, \text{ and} \quad (28)$$

$$I_{2-3} = -\frac{eq_-}{8a_0^4} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} \Big(r_-^3 - 4r_-^2\delta_- + 6r_-\delta_-^2 - 4\delta_-^3 + \frac{\delta_-^4}{r_-}\Big) \exp(-r_-/a_0)\, dr_-\,. \quad (29)$$

I_{2-1} is the sum of the following integrals:

$$I_{2-11} = -\frac{eq_-}{8a_0^3} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 4r_- \exp(-r_-/a_0)\, dr_- = -\frac{eq_-}{2a_0} - \frac{eq_-\delta_-}{2a_0^2}, \text{ and} \quad (30)$$

$$I_{2-12} = \frac{eq_-}{8a_0^3} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 8\delta_- \exp(-r_-/a_0)\, dr_- = +\frac{eq_-\delta_-}{a_0^2}\,. \quad (31)$$

So $I_{2-1} = -eq_-/2a_0 + eq_-\delta_-/2a_0^2$.

I_{2-2} is the sum of the following integrals:

$$I_{2-21} = \frac{eq_-}{8a_0^4} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 4r_-^2 \exp(-r_-/a_0)\, dr_- = +\frac{eq_-}{a_0} + \frac{eq_-\delta_-}{a_0^2}, \text{ and} \quad (32)$$

$$I_{2-22} = -\frac{eq_-}{8a_0^3} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} 12\delta_- r_- \exp(-r_-/a_0)\, dr_- = -\frac{3eq_-\delta_-}{2a_0^2}\,. \quad (33)$$

So $I_{2-2} = +eq_-/a_0 - eq_-\delta_-/2a_0^2$.

I_{2-3} is the sum of the following integrals:

$$I_{2-31} = -\frac{eq_-}{8a_0^5} \exp(+\delta_-/a_0) \int_{\delta_-}^{\infty} r_-^3 \exp(-r_-/a_0)\, dr_- = -\frac{3eq_-}{4a_0} - \frac{3eq_-\delta_-}{4a_0^2}, \text{ and} \quad (34)$$

$$I_{2-32} = +\frac{eq_-}{8a_0^3}\exp(+\delta_-/a_0)\int_{\delta_-}^{\infty} 4r_-^2\,\delta_-\exp(-r_-/a_0)\,dr_- = +\frac{eq_+\delta_-}{a_0^2}\,. \quad (35)$$

So $I_{2-3} = -3eq_-/(4a_0) + eq_-\delta_-/(4a_0^2)$.

Adding all the contributions to the energy shift from a down quark yields $\delta E_- = +eq_-\delta_-/(4a_0^2)$.

11 TOTAL ENERGY SHIFT

The total energy shift of the $2S$ level is the sum of $2\delta E_+$ and δE_-, so

$$\delta E(2S) = -\frac{2eq_+\delta_+}{4a_0^2} + \frac{eq_-\delta_-}{4a_0^2}\,. \quad (36)$$

Let m_+ be the mass of an up quark, and let m_- be the mass of the down quark. Take $m_- = 2m_+$(this is approximate). With the center of mass of the quarks at the origin, $2m_+\delta_+ = m_-\delta_-$, so $\delta_- = \delta_+$. Substitute $q_+ = 2e/3$ and $q_- = -e/3$ into Eq. (36), and find

$$\delta E(2S) = -\frac{4e^2\delta_+}{12a_0^2} - \frac{e^2\delta_+}{12a_0^2} = -\frac{5}{12}\frac{e^2\delta_+}{a_0^2}\,. \quad (37)$$

In future papers, the effect on the hydrogen atom spectrum of both quark displacement as in the model and proton size will be studied.

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