QUARK EFFECT ON MUONIC HYDROGEN SPECTRUM

May 7, 2018, revised November 2, 2018

1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the muonic hydrogen spectrum. There are three quarks (point particles) making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $-e$ is the electron charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$. When only the proton is present, take all three quarks to be at the center of the proton.

2 QUARK MODEL AND THE HYDROGEN ATOM

For a proton in the hydrogen atom, it will be assumed that the electron attracts the positively charged quarks, and repels the negatively charged quark. The positive quarks will be assumed to be displaced from the origin on the line from the origin to the electron. Label the displacement of the up quarks from the origin δ_+. The negative quark is assumed to be on the extension of the line from the electron through the origin. Label that displacement from the origin δ_-. In the first two papers cited below, it is argued that $\delta_- \approx \delta_+$.

The displacements of the quarks lead to an energy shift of the $2P$ energy level of $\Delta E_e(2P) = -5e^2\delta_+/(36a_0^2)$,[1] and also an energy shift of the $2S$ energy level of $\Delta E_e(2S) = -5e^2\delta_+/(12a_0^2)$.[2] Here a_0 is the Bohr radius. The difference in the energy shifts is $\Delta E_e = \Delta E_e(2P) - \Delta E_e(2S) = +5e^2\delta_+/(18a_0^2)$. It will be assumed that δ_+/a_0 is so small that ΔE_e has negligible affect on the hydrogen atom spectrum. So the experimentally determined value $r_p = 0.8768\,\text{fm}$ will be accepted as the root mean square proton radius.[3]

3 LAMB SHIFT IN MUONIC HYDROGEN[4,5,6]

The difference between the $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ energy levels of muonic hydrogen has been experimentally measured to be $\Delta E_\mu = 206.2949\,\text{meV}$. The theoretical difference is $\Delta E_\mu = 209.9779\,\text{meV-}5.2262\,r_p^2 + 0.0347\,r_p^3$ where r_p is the root mean square proton radius in fm. Equate the measured ΔE_μ to the theoretical ΔE_μ , and find $r_p = 0.84184\,\text{fm}$.

4 QUARK MODEL AND MUONIC HYDROGEN

Apply the polarization of quarks described in sections (1) and (2) to muonic hydrogen. The energy shift ΔE_μ takes the same form as ΔE_e, but a_0 is replaced by the muon Bohr radius $a_{0\mu}$. The mass of the muon is approximately 200 times the mass of the electron, so $a_{0\mu}$ will be about 200 times smaller than a_0. Thus the muon will be closer to the quarks than the electron, and as a result, δ_+ will be larger in muonic hydrogen than in hydrogen. In addition, $1/a_{0\mu}^2 \approx 200^2/a_0^2$, so ΔE_μ is much larger than ΔE_e.

Refer back to section (3). Equate the measured energy difference between the two levels to the theoretical energy difference as in section (3), but add ΔE_μ to the theoretical splitting. Thus

$$206.2949\,\text{meV} = 209.9779\,\text{meV} - 5.2262\,r_p^2 + 0.0347\,r_p^3 + 5e^2\delta_+/(18a_{0\mu}^2). \quad (1)$$

Substitute the accepted value $r_p = 0.8768\,\text{fm}$ into the above equation, and solve for δ_+. The result is $\delta_+ \approx 5.45 \cdot 10^{-20}$ meters, which shows that a small polarization can have a significant effect on the spectrum of muonic hydrogen.

Thus the polarization of the quarks caused by the presence of the muon provides a possible solution to the current proton radius puzzle. However, the model is too simple to be conclusive. It is hoped that a more realistic quark model will confirm the conclusions of this paper.

ACKNOWLEDGMENTS

I thank Ben for his insightful suggestions.

References

[1] http://www.electronformfactor.com,/Quark Effect On H-Atom Spectra(2P)

[2] http://www.electronformfactor.com/Quark Effect On H-Atom Spectra(2S)

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[4] Pohl R, Antognini A, Nez F, Amaro FD, Biraben F, et al., Nature 466:213 (2010)

[5] Antognini A, Nez F, Schuhmann K, Amaro FD, Biraben F, et al., Science 339:417 (2013).

[6] http://www.fuw.edu.pl/ krp/papers/pohl.pdf