EXTENDED QUARK EFFECT ON HYDROGEN SPECTRUM(2P)

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectra. There are three quarks making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $-e$ is the electron charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$.

The quark charge is assumed to be uniformly distributed on a spherical shell of radius r_o. Thus the charge probability density of each quark is assumed to be $q_\pm \delta(y - r_o)/(4\pi r_0^2)$ where y is the distance from the quark center to a spherical shell of radius r_o and $\delta()$ is the Dirac delta function. Take the proton to be at the origin, and the proton radius to also be r_o. When only the proton is present, take the three quark centers to also be at the origin. . For a proton in the presence of an electron, it will be assumed that the electron attracts the positively charged quarks, and repels the negatively charged quark. When the electron is a distance r from the origin, assume the up quark is displaced by a distance δ_+. As the electron moves closer to the origin, it is expected that δ_+ would increase. In the interest of mathematical simplicity, take δ_+ to be constant. So a positive quark is always assumed to be displaced from the origin on the line from the origin to the electron by a constant distance δ_+ toward the electron. Similarly, the negative quark is always assumed to be displaced from the origin on the extension of the line from the electron through the origin by a constant distance δ_-. The displacements δ_+ and δ_- will be assumed to be much smaller than both a_0, the Bohr radius and r_o, so $\delta_\pm << r_o << a_0$.

Often the Taylor expansion $\exp(-\delta_\pm/a_0) = 1 - \delta_\pm/a_0 + \delta_\pm^2/(2a_0^2) + \cdots$ will enter the equations for the energy shifts. Only terms in the energy shifts like $eq_\pm/a_0$ and

$eq_\pm\delta_\pm/a_0^2$ will be kept. Terms such as $eq_\pm\delta_\pm^n/a_0^{n+1}$ for $n \geq 2$ are negligible in comparison, and will be discarded.

2 POTENTIAL ENERGY OF AN UP QUARK AND A POINT ELECTRON [1]

Let $\mathbf{r}_+$ be the vector from the center of the quark to the electron, and let r_+ be the corresponding distance. Let $\mathbf{y}$ be the vector from the quark center to an element of quark charge, and let y be the corresponding distance. Define $\mathbf{r}'$ to be the vector from an element of quark charge to the electron, and r' is the corresponding distance. The three vectors form a triangle, and $r'^2 = r_+^2 + y^2 - 2r_+y\cos(\theta)$ where θ is the angle between $\mathbf{r}_+$ and $\mathbf{y}$. The differential potential of the quark is

$$d\Phi_+ = \frac{q_+}{r'}\frac{\delta(y - r_o)}{4\pi r_o^2}y^2 \sin(\theta)\, d\theta\, d\phi\, dy \qquad (1)$$

Set $u = -\cos(\theta)$. Then

$$\Phi_+ = q_+\int_{-1}^{+1}\frac{du}{2\sqrt{r_+^2 + r_o^2 + 2r_+r_o\, u}} = \frac{q_+}{r_+}H(r_+ - r_o) + \frac{q_+}{r_o}H(r_o - r_+) \qquad (2)$$

where $H()$ is the unit step function.

Let r denote the distance from he origin to the electron. Then $r = r_+ + \delta_+$, and the potential energy of a top quark and the electron is conveniently written

$$V_+ = -\frac{eq_+}{r_+}H(r_+ - r_o) - \frac{eq_+}{r_o}H(r_o + \delta_+ - r)\,. \qquad (3)$$

3 POTENTIAL ENERGY OF A DOWN QUARK AND THE ELECTRON

Let $\mathbf{r}_-$ be the vector from the center of the quark to the electron, and let r_- be the corresponding distance. Again, let $\mathbf{r}'$ be the vector from an element of quark charge to the electron. Then $r'^2 = r_-^2 + y^2 - 2r_-y\cos(\theta)$ where θ is the angle between $\mathbf{r}_-$ and $\mathbf{y}$. The differential potential of the quark is

$$d\Phi_- = \frac{q_-}{r'}\frac{\delta(y - r_o)}{4\pi r_o^2}y^2 \sin(\theta)\, d\theta\, d\phi\, dy \qquad (4)$$

Set $u = -\cos(\theta)$. Then

$$\Phi_- = q_- \int_{-1}^{+1} \frac{du}{2\sqrt{r_-^2 + r_o^2 + 2r_- r_o\, u}} = \frac{q_-}{r_-} H(r_- - r_o) + \frac{q_-}{r_o} H(r_o - r_-)\,. \quad (5)$$

Since $r_- = r + \delta_-$, the potential energy of the down quark and the electron is conveniently written

$$V_- = -\frac{eq_-}{r_-} H(r_- - r_o) - \frac{eq_-}{r_o} H(r_o - \delta_- - r)\,. \quad (6)$$

4 THE HAMILTONIAN

For the proton and quarks at the origin, the unperturbed Hamiltonian of an up quark and the electron is $H_{o+} = p^2/2\mu_+ - eq_+/r$ where μ_+ is the reduced mass of an up quark and the electron. The Hamiltonian of an up quark displaced by the distance δ_+ and an electron is

$$H_+ = \frac{p^2}{2\mu_+} + V_+ = \frac{p^2}{2\mu_+} - \frac{eq_+}{r} + \frac{eq_+}{r} + V_+ = H_{o+} + H'_+ \quad (7)$$

where the perturbing Hamiltonian for an up quark and the electron is

$$H'_+ = +\frac{eq_+}{r} - \frac{eq_+}{r_+} H(r_+ - r_o) - \frac{eq_+}{r_o} H(r_o + \delta_+ - r) \quad (8)$$

The unperturbed Hamiltonian of the down quark and the electron is given by $H_{o-} = p^2/2\mu_- - eq_-/r$ where μ_- is the reduced mass of the down quark and the electron. The Hamiltonian for the down quark displaced by the distance δ_- and an electron is

$$H_- = \frac{p^2}{2\mu_-} + V_- = \frac{p^2}{2\mu_-} - \frac{eq_-}{r} + \frac{eq_-}{r} + V_- = H_{o-} + H'_- \quad (9)$$

where the perturbing Hamiltonian for the down quark is

$$H'_- = \frac{eq_-}{r} - \frac{eq_-}{r_-} H(r_- - r_o) - \frac{eq_-}{r_o} H(r_o - \delta_- - r)\,. \quad (10)$$

So the perturbing Hamiltonian for the quarks and the electron is $H' = 2H'_+ + H'_-$.

5 ENERGY SHIFT DUE TO H'_+

The energy shift associated with H'_+ is

$$\delta E_+ = \int \psi^*(r,\theta,\phi) H'_+ \psi(r,\theta,\phi)\, d^3r = I_{1+} + I_{2+} + I_{3+} \quad (11)$$

where $d^3r = r^2\,\sin(\theta)\,d\theta\,d\phi\,dr$, the unperturbed wave function for the $2P$ energy level is $\psi(r,\theta,\phi) = r\exp(-r/(2a_0)Y_{1,m}(\theta,\phi)/\big(\sqrt{(2a_0)^3}(\sqrt{3}a_0)\big)$, $Y_{1,m}(\theta,\phi)$ is a normalized spherical harmonic,

$$I_{1+} = \int \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}|Y_{1,m}|^2\Big(\frac{eq_+}{r}\Big)r^2\,\sin(\theta)\,d\theta\,d\phi\,dr = \frac{eq_+}{4a_0}\,, \tag{12}$$

$$I_{2+} = -\int \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_+}{r_+}H(r_+ - r_o)\Big)r^2\,dr\,,\ \text{and} \tag{13}$$

$$I_{3+} = -\int \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_+}{r_o}H(r_o + \delta_+ - r)\Big)\,r^2\,dr\,. \tag{14}$$

6 CALCULATION OF I_{2+}

Substitute $r = r_+ + \delta_+$ in Eq. (13). Then

$$I_{2+} = -\frac{eq_+}{(2a_0)^3}\frac{\exp(-\delta_+/a_0)}{3a_0^2}\int_{r_o}^{\infty}\exp(-r_+/a_0)\Big(r_+^3 + 4r_+^2\delta_+ + 6r_+\delta_+^2 + 4\delta_+^3 + \frac{\delta_+^4}{r_+}\Big)dr_+\,. \tag{15}$$

I_{2+} is the sum of the following integrals:

$$\begin{aligned} I_{2+1} = &-\frac{eq_+}{(2a_0)^3}\frac{\exp(-\delta_+/a_0)}{3a_0^2}\int_{r_o}^{\infty} r_+^3\exp(-r_+/a_0)dr_+ = \\ &-\frac{eq_+}{(2a_0)^3}\frac{\exp(-\delta_+/a_0)}{3a_0^2}\exp(-r_+/a_0)\Big(-r_+^3a_0 - 3r_+^2a_0^2 - 6r_+a_0^3 - 6a_0^4\Big)\Big|_{r_o}^{\infty} = \\ &-\frac{eq_+}{(2a_0)^3}\Big(1 - \frac{r_o+\delta_+}{a_0} + \frac{(r_o+\delta_+)^2}{2a_0^2}\Big)\Big(r_o^2 + 2r_oa_0 + 2a_0^2\Big) = \\ &-\frac{eq_+}{4a_0} + \frac{eq_+\delta_+}{4a_0^2}\,,\text{and} \end{aligned} \tag{16}$$

$$\begin{aligned} I_{2+2} = &-\frac{eq_+}{(2a_0)^3}\frac{\exp(-\delta_+/a_0)}{3a_0^2}4\delta_+\int_{r_o}^{\infty} r_+^2\exp(-r_+/a_0)dr_+ = \\ &-\frac{eq_+}{(2a_0)^3}\frac{\exp(-\delta_+/a_0)}{3a_0^2}4\delta_+\exp(-r_+/a_0)\Big(-r_+^2a_0 - 2r_+a_0^2 - 2a_0^3\Big)\Big|_{r_o}^{\infty} = \\ &-4\frac{eq_+\delta_+}{(2a_0)^3}\Big(1 - \frac{r_o+\delta_+}{a_0} + \frac{(r_o+\delta_+)^2}{2a_0^2}\Big)\Big(\frac{2r_o}{3} + \frac{2a_0}{3}\Big) = -\frac{eq_+\delta_+}{3a_0^2}\,. \end{aligned} \tag{17}$$

The last three terms are discarded because they contain δ_+^n for $n \geq 2$.

Add the results together, and find $I_{2+} = -eq_+/4a_0 - eq_+\delta_+/(12a_0^2)$.

7 CALCULATION OF I_{3+}

Eq. (14) takes the form

$$I_{3+} = -\frac{eq_+}{(2a_0)^3}\frac{1}{3a_0^2 r_o}\int_0^{r_o+\delta_+} r^4 \exp(-r/a_0)\, dr = \\ -\frac{eq_+}{(2a_0)^3}\frac{1}{3a_0^2 r_o}\int_0^{r_o+\delta_+} r^4\, dr = -\frac{eq_+}{(2a_0)^3}\frac{1}{3a_0^2 r_o}\frac{r^5}{5}\Big|_0^{r_o+\delta_+} \quad (18)$$

This term will be discarded since it contains high powers of a_0 in the denominator.
Add Eq. (12), Eq. (16), and Eq. (17), and find $\delta E_+ = -eq_+\delta_+/(12a_0^2)$

8 ENERGY SHIFT DUE TO H'_-

The energy shift associated with H'_- is

$$\delta E_- = \int \psi^*(r,\theta,\phi) H'_- \psi(r,\theta,\phi)\, d^3r = I_{1-} + I_{2-} + I_{3-} \quad (19)$$

where

$$I_{1-} = \int_0^\infty \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_-}{r}\Big) r^2\, dr = \frac{eq_-}{4a_0}\,, \quad (20)$$

$$I_{2-} = -\int \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_-}{r_-} H(r_- - r_o)\Big) r^2\, dr\,, \text{ and} \quad (21)$$

$$I_{3-} = -\int \frac{r^2}{3a_0^2}\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_-}{r_o} H(r_o - \delta_- - r)\Big) r^2\, dr\,. \quad (22)$$

9 CALCULATION OF I_{2-}

Substitute $r = r_- - \delta_-$ in Eq. (21). Then

$$I_{2-} = -\frac{eq_-}{(2a_0)^3}\frac{\exp(+\delta_-/a_0)}{3a_0^2}\int_{r_o}^\infty \exp(-r_-/a_0)\Big(r_-^3 - 4r_-^2\delta_- + 6r_-\delta_-^2 + 4\delta_-^3 - \frac{\delta_-^4}{r_-}\Big) dr_-\,. \quad (23)$$

After discarding terms in high powers of δ_-, I_{2-} is the sum of the following integrals:

$$I_{2-1} = -\frac{eq_-}{(2a_0)^3}\frac{\exp(+\delta_-/a_0)}{3a_0^2}\int_{r_o}^{\infty}\exp(-r_-/a_0)r_-^3\,dr_- =$$
$$-\frac{eq_-}{(2a_0)^3}\frac{\exp(+\delta_-/a_0)}{3a_0^2}\exp(-r_-/a_0))\Big(-r_-^3a_0 - 3r_-^2a_0^2 - 6r_-a_0^3 - 6a_0^4\Big)\Big|_{r_o}^{\infty} =$$
$$-\frac{eq_-}{(2a_0)^3}\Big(1 - \frac{(r_o-\delta_-)}{a_0} + \frac{(r_o-\delta_-)^2}{2a_0^2}\Big)\Big(r_o^2 - 2r_oa_0 + 2a_0^2\Big) =$$
$$-\frac{eq_-}{4a_0} - \frac{eq_-\delta_-}{4a_0^2}\text{, and} \quad (24)$$

$$I_{2-2} = +\frac{eq_-}{(2a_0)^3}\frac{\exp(+\delta_-/a_0)}{3a_0^2}4\delta_-\int_{r_o}^{\infty}r_-^2\exp(-r_-/a_0)dr_- =$$
$$+\frac{e_-}{(2a_0)^3}\frac{\exp(+\delta_-/a_0)}{3a_0^2}4\delta_-\exp(-r_-/a_0)\Big(-r_-^2a_0 - 2r_-a_0^2 - 2a_0^3\Big)\Big|_{r_o}^{\infty} =$$
$$+4\frac{eq_-\delta_-}{(2a_0)^3}\Big(1 - \frac{r_o-\delta_-}{a_0} + \frac{(r_o-\delta_-)^2}{2a_0^2}\Big)\Big(\frac{2r_o}{3} + \frac{2a_0}{3}\Big) = +\frac{eq_-\delta_-}{3a_0^2}. \quad (25)$$

The last three terms are discarded as they contain high powers of δ_-. Add the results, and get $I_{2-} = -eq_-/(4a_0) + eq_-\delta_-/(12a_0^3)$.

10 CALCULATION OF I_{3-}

Eq. (22) takes the form

$$I_{3-} = -\frac{eq_-}{(2a_0)^3}\frac{1}{3a_0^2r_o}\int_0^{r_o-\delta_-}r^4\exp(-r/a_0)\,dr =$$
$$-\frac{eq_-}{(2a_0)^3}\frac{1}{3a_0^2r_o}\int_0^{r_o-\delta_-}r^4\,dr = -\frac{eq_-}{(2a_0)^3}\frac{1}{3a_0^2r_o}\frac{r^5}{5}\Big|_0^{r_o-\delta_-} \quad (26)$$

This term will be discarded since it contains high powers of a_0 in the denominator.

Add Eq. (20), Eq. (24) and Eq. (25) , and find $\delta E_- = +eq_-\delta_-/(12a_0^2)$

11 ENERGY SHIFT OF THE $2P$ LEVEL

The total energy shift of the $2P$ level is the sum of $2\delta E_+$ and δE_-, so

$$\delta E(2P) = 2\Big(-\frac{eq_+\delta_+}{12a_0^2}\Big) + \frac{eq_-\delta_-}{12a_0^2} \quad (27)$$

Let m_+ be the mass of an up quark, and let m_- be the mass of the down quark. Take $m_- = 2m_+$(this is approximate). With the center of mass of the quarks at the origin, $2m_+\delta_+ = m_-\delta_-$, so $\delta_- = \delta_+$. Substitute $q_+ = 2e/3$ and $q_- = -e/3$ into Eq. (27), and find

$$\delta E(2P) = -\frac{4e^2\delta_+}{36a_0^2} - \frac{e^2\delta_+}{36a_0^2} = -\frac{5e^2\delta_+}{36a_0^2}\,. \quad (28)$$

The energy shift due to proton size is known to be $\delta E = 2\pi e^2 r_p^2 |\psi(r=0)|^2/3$ where r_p^2 is the mean square radius of the proton.[2] For the $2P$ level, $\psi(r=0) = 0$, so the r_p^2 term does not appear. Note that for the chosen quark charge density, $r_o = r_p$.

ACKNOWLEDGMENTS

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References

[1] http://www.electronformfactor.com/Extended Quark Effect On Hydrogen Spectrum(1S).

[2] http://www.fuw.edu.pl/ krp/papers/pohl.pdf