EXTENDED QUARK EFFECT ON HYDROGEN SPECTRA(2S)

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectra. There are three quarks making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $-e$ is the electron charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$.

The quark charge is assumed to be uniformly distributed on a spherical shell of radius r_o. Thus the charge probability density of each quark is assumed to be $q_\pm \delta(y - r_o)/(4\pi r_0^2)$ where y is the distance from the quark center to a spherical shell of radius r_o and $\delta()$ is the Dirac delta function. Take the proton to be at the origin, and the proton radius to also be r_o. When only the proton is present, take the three quark centers to also be at the origin. .

For a proton in the presence of an electron, it will be assumed that the electron attracts the positively charged quarks, and repels the negatively charged quark. When the electron is a distance r from the origin, assume the up quark is displaced by a distance δ_+. As the electron moves closer to the origin, it is expected that δ_+ would increase. In the interest of mathematical simplicity, take δ_+ to be constant. So a positive quark is always assumed to be displaced from the origin on the line from the origin to the electron by a constant distance δ_+ toward the electron. Similarly, the negative quark is always assumed to be displaced from the origin on the extension of the line from the electron through the origin by a constant distance δ_-. The displacements δ_+ and δ_- will be assumed to be much smaller than both a_0, the Bohr radius and r_o, so $\delta_\pm << r_o << a_0$.

Often the Taylor expansion $\exp(-\delta_\pm/a_0) = 1 - \delta_\pm/a_0 + \delta_\pm^2/(2a_0^2) + \cdots$ will enter the equations for the energy shifts. Only terms in the energy shifts like $eq_\pm/a_0$, $eq_\pm r_o^2/a_0^3$, and $eq_\pm\delta_\pm/a_0^2$ will be kept. Terms such as $eq_\pm\delta_\pm^n/a_0^{n+1}$ for $n \geq 2$ are negligible in comparison, and will be discarded.

2 POTENTIAL ENERGY OF AN UP QUARK AND A POINT ELECTRON [1]

Let $\mathbf{r}_+$ be the vector from the center of the quark to the electron, and let r_+ be the corresponding distance. Let $\mathbf{y}$ be the vector from the quark center to an element of quark charge, and let y be the corresponding distance. Define $\mathbf{r}'$ to be the vector from an element of quark charge to the electron, and r' is the corresponding distance. The three vectors form a triangle, and $r'^2 = r_+^2 + y^2 - 2r_+y\cos(\theta)$ where θ is the angle between $\mathbf{r}_+$ and $\mathbf{y}$. The differential potential of the quark is

$$d\Phi_+ = \frac{q_+}{r'}\frac{\delta(y - r_o)}{4\pi r_o^2} y^2 \,\sin(\theta)\, d\theta\, d\phi\, dy \tag{1}$$

Set $u = -\cos(\theta)$. Then

$$\Phi_+ = q_+ \int_{-1}^{+1} \frac{du}{2\sqrt{r_+^2 + r_o^2 + 2r_+r_o\, u}} = \frac{q_+}{r_+}H(r_+ - r_o) + \frac{q_+}{r_o}H(r_o - r_+) \tag{2}$$

where $H()$ is the unit step function.

Let r denote the distance from he origin to the electron. Then $r = r_+ + \delta_+$, and the potential energy of a top quark and the electron is conveniently written

$$V_+ = -\frac{eq_+}{r_+}H(r_+ - r_o) - \frac{eq_+}{r_o}H(r_o + \delta_+ - r)\,. \tag{3}$$

3 POTENTIAL ENERGY OF A DOWN QUARK AND THE ELECTRON

Let $\mathbf{r}_-$ be the vector from the center of the quark to the electron, and let r_- be the corresponding distance. Again, let $\mathbf{r}'$ be the vector from an element of quark charge to the electron. Then $r'^2 = r_-^2 + y^2 - 2r_-y\cos(\theta)$ where θ is the angle between $\mathbf{r}_-$ and $\mathbf{y}$. The differential potential of the quark is

$$d\Phi_- = \frac{q_-}{r'}\frac{\delta(y - r_o)}{4\pi r_o^2} y^2 \,\sin(\theta)\, d\theta\, d\phi\, dy \tag{4}$$

Set $u = -\cos(\theta)$. Then

$$\Phi_- = q_- \int_{-1}^{+1} \frac{du}{2\sqrt{r_-^2 + r_o^2 + 2r_- r_o\, u}} = \frac{q_-}{r_-} H(r_- - r_o) + \frac{q_-}{r_o} H(r_o - r_-)\,. \quad (5)$$

Since $r_- = r + \delta_-$, the potential energy of the down quark and the electron is conveniently written

$$V_- = -\frac{eq_-}{r_-} H(r_- - r_o) - \frac{eq_-}{r_o} H(r_o - \delta_- - r)\,. \quad (6)$$

4 THE HAMILTONIAN

For the proton and quarks at the origin, the unperturbed Hamiltonian of an up quark and the electron is $H_{o+} = p^2/2\mu_+ - eq_+/r$ where μ_+ is the reduced mass of an up quark and the electron. The Hamiltonian of an up quark displaced by the distance δ_+ and an electron is

$$H_+ = \frac{p^2}{2\mu_+} + V_+ = \frac{p^2}{2\mu_+} - \frac{eq_+}{r} + \frac{eq_+}{r} + V_+ = H_{o+} + H'_+ \quad (7)$$

where the perturbing Hamiltonian for an up quark and the electron is

$$H'_+ = +\frac{eq_+}{r} - \frac{eq_+}{r_+} H(r_+ - r_o) - \frac{eq_+}{r_o} H(r_o + \delta_+ - r) \quad (8)$$

The unperturbed Hamiltonian of the down quark and the electron is given by $H_{o-} = p^2/2\mu_- - eq_-/r$ where μ_- is the reduced mass of the down quark and the electron. The Hamiltonian for the down quark displaced by the distance δ_- and an electron is

$$H_- = \frac{p^2}{2\mu_-} + V_- = \frac{p^2}{2\mu_-} - \frac{eq_-}{r} + \frac{eq_-}{r} + V_- = H_{o-} + H'_- \quad (9)$$

where the perturbing Hamiltonian for the down quark is

$$H'_- = \frac{eq_-}{r} - \frac{eq_-}{r_-} H(r_- - r_o) - \frac{eq_-}{r_o} H(r_o - \delta_- - r)\,. \quad (10)$$

So the perturbing Hamiltonian for the quarks and the electron is $H' = 2H'_+ + H'_-$.

5 ENERGY SHIFT DUE TO H'_+

The energy shift associated with H'_+ is

$$\delta E_+ = \int \psi^*(r,\theta,\phi) H'_+ \psi(r,\theta,\phi)\, d^3r = I_{1+} + I_{2+} + I_{3+} \quad (11)$$

where $d^3r = r^2\,\sin(\theta)\,d\theta\,d\phi\,dr$, the unperturbed wave function for the $2S$ energy level is $\psi(r,\theta,\phi) = (2-r/a_0)\exp(-r/(2a_0)Y_{0,0}/\big(\sqrt{(2a_0)^3}\big)$, $Y_{0,0}(\theta,\phi) = 1/\sqrt{4\pi}$ is a normalized spherical harmonic,

$$I_{1+} = \int \Big(4 - 4\frac{r}{a_0} + \frac{r^2}{a_0^2}\Big)\frac{\exp(-r/a_0)}{(2a_0)^3 4\pi}\Big(\frac{eq_+}{r}\Big) r^2\,\sin(\theta)\,d\theta\,d\phi\,dr = \frac{eq_+}{4a_0}\,, \quad (12)$$

$$I_{2+} = -\int \Big(4 - 4\frac{r}{a_0} + \frac{r^2}{a_0^2}\Big)\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_+}{r_+}H(r_+ - r_o)\Big) r^2\,dr\,, \text{ and} \quad (13)$$

$$I_{3+} = -\int \Big(4 - 4\frac{r}{a_0} + \frac{r^2}{a_0^2}\Big)\frac{\exp(-r/a_0)}{(2a_0)^3}\Big(\frac{eq_+}{r_o}H(r_o + \delta_+ - r)\Big)\, r^2\,dr\,. \quad (14)$$

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6 CALCULATION OF I_{2+}

Substitute $r = r_+ + \delta_+$ in Eq. (13). After dropping terms containing δ_+^n for $n \geq 2$,

$$I_{2+} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty}\exp(-r_+/a_0)$$
$$\Big[4r_+ + 8\delta_+\delta_+ - 4\frac{r_+^2}{a_0} - 12\frac{r_+\delta_+}{a_0} + \frac{r_+^3}{a_0^2} + \frac{4r_+^2\delta_+}{a_0^2}\Big]dr_+\,. \quad (15)$$

I_{2+} is the sum of the following integrals:

$$I_{2+1} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty} 4r_+\exp(-r_+/a_0)dr_+ =$$
$$-4\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\exp(-r_+/a_0)(-r_+a_0 - a_0^2)\Big|_{r_o}^{\infty} =$$
$$-4\frac{eq_+}{(2a_0)^3}\Big(1 - \frac{r_o + \delta_+}{a_0} + \frac{(r_o+\delta_+)^2}{2a_0^2}\Big)\Big(r_o a_0 + a_0^2\Big) =$$
$$-\frac{eq_+}{2a_0} + \frac{eq_+ r_o^2}{4a_0^3} + \frac{eq_+\delta_+}{2a_0^2}\,, \quad (16)$$

$$I_{2+2} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)8\delta_+\int_{r_o}^{\infty}\exp(-r_+/a_0)dr_+ =$$
$$-\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)8\delta_+\exp(-r_+/a_0)(-a_0)\Big|_{r_o}^{\infty} =$$
$$-\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)8\delta_+\exp(-r_o/a_0)a_0 = -\frac{eq_+\delta_+}{a_0^2} \quad (17)$$

$$I_{2+3} = +\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty} 4\frac{r_+^2}{a_0}\exp(-r_+/a_0)dr_+ =$$
$$+\,4\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\exp(-r_o/a_0)\Big(\frac{r_o^2a_0 + 2r_oa_0^2 + 2a_0^3)}{a_0}\Big) =$$
$$\frac{eq_+}{a_0} - \frac{eq_+\delta_+}{a_0^2}\,, \quad (18)$$

$$I_{2+4} = \frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty} 12\frac{r_+\delta_+}{a_0}\exp(-r_+/a_0)dr_+ =$$
$$12\frac{eq_+\delta_+}{(2a_0)^3}\exp(-\delta_+/a_0)\exp(-r_o/a_0)\Big(\frac{r_oa_0 + a_0^2}{a_0}\Big) = \frac{3eq_+\delta_+}{2a_0^2}\,, \quad (19)$$

$$I_{2+5} = -\frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty}\frac{r_+^3}{a_0^2}\exp(-r_+/a_0)dr_+ =$$
$$-\frac{eq_+\delta_+}{(2a_0)^3}\exp(-r_+/a_0 - \delta_+/a_0)\Big(\frac{r_o^3a_0 + 3r_o^2a_0^2 + 6r_oa_0^3 + 6a_0^4}{a_0^2}\Big) = -\frac{3eq_+}{4a_0} + \frac{3eq_+\delta_+}{4a_0^2} \text{ and,} \quad (20)$$

$$I_{2+6} = \frac{eq_+}{(2a_0)^3}\exp(-\delta_+/a_0)\int_{r_o}^{\infty}\frac{4r_+^2\delta_+}{a_0^2}\exp(-r_+/a_0)dr_+ =$$
$$\frac{4eq_+\delta_+}{(2a_0)^3}\exp(-r_+/a_0 - \delta_+/a_0)\Big(\frac{r_o^2a_0 + 2r_oa_0^2 + 2a_0^3}{a_0^2}\Big) = -\frac{eq_+\delta_+}{a_0^2}\,. \quad (21)$$

Add the results together, and find

$$I_{2+} = -\frac{eq_+}{4a_0} + \frac{eq_+r_o^2}{4a_0^3} - \frac{eq_+\delta_+}{4a_0^2}\,. \quad (22)$$

7 CALCULATION OF I_{3+} AND δE_+

Eq. (14) takes the form

$$I_{3+} = -\frac{eq_+}{(2a_0)^3r_o}\int_0^{r_o+\delta_+}\Big(4r^2 - 4\frac{r^3}{a_0} + \frac{r^4}{a_0^2}\Big)\exp(-r/a_0)\,dr =$$
$$-\,4\frac{eq_+}{(2a_0)^3r_o}\int_0^{r_o+\delta_+} r^2\,dr = -\frac{eq_+r_o^2}{(6a_0)^3} \quad (23)$$

Note that terms with high powers of a_0 in the denominator have been discarded.

So $\delta E_+ = I_{1+} + I_{2+} + I_{3+} = +eq_+r_o^2/(12a_0^3) - eq_+\delta_+/(4a_0^2)$.

8 ENERGY SHIFT DUE TO H'_-

The energy shift associated with H'_- is

$$\delta E_- = \int \psi^*(r,\theta,\phi) H'_- \psi(r,\theta,\phi)\, d^3r = I_{1-} + I_{2-} + I_{3-} \tag{24}$$

where

$$I_{1-} = \int_0^\infty \Big(4r^2 - 4\frac{r^3}{a_0} + \frac{r^4}{a_0^2}\Big) \frac{\exp(-r/a_0)}{(2a_0)^3} \Big(\frac{eq_-}{r}\Big)\, dr = \frac{eq_-}{4a_0}\,, \tag{25}$$

$$I_{2-} = -\int \Big(4r^2 - 4\frac{r^3}{a_0} + \frac{r^4}{a_0^2}\Big) \frac{\exp(-r/a_0)}{(2a_0)^3} \Big(\frac{eq_-}{r_-} H(r_- - r_o\Big)\, dr\,, \text{ and} \tag{26}$$

$$I_{3-} = -\int \Big(4r^2 - 4\frac{r^3}{a_0} + \frac{r^4}{a_0^2}\Big) \frac{\exp(-r/a_0)}{(2a_0)^3} \Big(\frac{eq_-}{r_o} H(r_o - \delta_- - r)\Big)\, dr\,. \tag{27}$$

9 CALCULATION OF I_{2-}

Substitute $r = r_- - \delta_-$ in Eq. (26). After discarding terms in high powers of δ_-,

$$I_{2-} = -\frac{eq_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{r_o}^\infty \exp(-r_-/a_0) \Big(4r_- - 8\delta_- - \frac{(4r_-^2 - 12r_-\delta_-)}{a_0} + \frac{(r_-^3 - 4r_-^2\delta_-)}{a_0^2}\Big) dr_-\,. \tag{28}$$

I_{2-} is the sum of the following integrals:

$$I_{2-1} = -\frac{4eq_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{r_o}^\infty r_- \exp(-r_-/a_0\, dr_- =$$

$$-4\frac{eq_-}{(2a_0)^3} \exp(-r_o/a_0 + \delta_-/a_0)(r_o a_0 + a_0^2) = -\frac{eq_-}{2a_0} + \frac{eq_- r_o^2}{4a_0^3} - \frac{eq_-\delta_-}{2a_0^2} \tag{29}$$

$$I_{2-2} = +\frac{8eq_-\delta_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{r_o}^\infty \exp(-r_-/a_0) dr_+ =$$

$$+\frac{8eq_-\delta_-}{8a_0^3} \exp[-(r_o - \delta_-)/a_0] a_0 = +\frac{eq_-\delta_-}{a_0^2}\,, \tag{30}$$

$$I_{2-3} = +\frac{4eq_-}{(2a_0)^3} \exp(+\delta_-/a_0) \int_{r_o}^\infty \frac{r_-^2}{a_0} \exp(-r_-/a_0) dr_+ =$$

$$+\frac{4eq_-}{8a_0^3} \exp[-(r_o - \delta_-)/a_0](r_o^2 a_0 + 2r_o a_0^2 + 2a_0^3) = +\frac{eq_-}{a_0} + \frac{eq_-\delta_-}{a_0^2}\,, \tag{31}$$

$$I_{2-4} = -\frac{12eq_-\delta_-}{(2a_0)^3}\exp(+\delta_-/a_0)\int_{r_o}^{\infty}\frac{r_-}{a_0}\exp(-r_-/a_0)dr_+ =$$
$$-\frac{12eq_-}{8a_0^3}\exp[-(r_o-\delta_-)/a_0](r_oa_0+a_0^2) = -\frac{3eq_-\delta_-}{2a_0^2}\,, \quad (32)$$

$$I_{2-5} = -\frac{eq_-}{(2a_0)^3}\exp(+\delta_-/a_0)\int_{r_o}^{\infty}\frac{r_-^3}{a_0^2}\exp(-r_-/a_0)dr_+ =$$
$$-\frac{eq_-}{8a_0^3}\exp[-(r_o-\delta_-)/a_0](r_o^3a_0+3r_o^2a_0^2+6r_oa_0^3+6a_0^4) = -\frac{3eq_-}{4a_0}-\frac{3eq_-\delta_-}{4a_0^2}\,, \quad (33)$$

$$I_{2-6} = -\frac{4eq_-\delta_-}{(2a_0)^3}\exp(+\delta_-/a_0)\int_{r_o}^{\infty}\frac{r_-^2}{a_0^2}\exp(-r_-/a_0)dr_+ =$$
$$-\frac{4eq_-}{8a_0^3}\exp[-(r_o-\delta_-)/a_0](r_o^2a_0+2r_o^2a_0^2+2a_0^3) = +\frac{eq_-\delta_-}{a_0^2}\,. \quad (34)$$

Add the results together and find

$$I_{2-} = -\frac{eq_-}{4a_0}+\frac{eq_-r_o^2}{4a_0^3}+\frac{eq_-\delta_-}{4a_0^2}\,. \quad (35)$$

10 CALCULATION OF I_{3-} AND δE_-

Eq. (27) takes the form

$$I_{3-} = -\frac{eq_-}{8a_0^3r_o}\int_0^{r_o-\delta_-}\Big(4r^2-4\frac{r^3}{a_0}+\frac{r^4}{a_0^2}\Big)\exp(-r/a_0)\,dr\,. \quad (36)$$

After discarding negligible terms,

$$I_{3-} = -\frac{eq_-}{8a_0^3r_o}\int_0^{r_o-\delta_-}4r^2\,dr = -\frac{eq_-r_o^2}{6a_0^3} \quad (37)$$

Add the results and find $\delta E_- = +eq_-r_o^2/(12a_0^3) + eq_-\delta_-/(4a_0^2)$

11 ENERGY SHIFT OF THE $2S$ LEVEL

The total energy shift of the $2S$ level is the sum of $2\delta E_+$ and δE_-, so

$$\delta E(2S) = 2\Big(\frac{eq_+r_o^2}{12a_0^3}-\frac{eq_+\delta_+}{4a_0^2}\Big)+\Big(\frac{eq_-r_o^2}{12a_0^3}+\frac{eq_-\delta_-}{4a_0^2}\Big) \quad (38)$$

Let m_+ be the mass of an up quark, and let m_- be the mass of the down quark. Take $m_- = 2m_+$(this is approximate). With the center of mass of the quarks at the origin, $2m_+\delta_+ = m_-\delta_-$, so $\delta_- = \delta_+$. Substitute $q_+ = 2e/3$ and $q_- = -e/3$ into Eq. (38), and find

$$\delta E(2S) = +\frac{e^2 r_o^2}{12a_0^3} - \frac{5e^2\delta_+}{12a_0^2}\,. \tag{39}$$

The energy shift due to proton size is known to be $\delta E_p = 2\pi e^2 r_p^2 |\psi(r=0)|^2/3$ where r_p^2 is the mean square radius of the proton.[2] So for the $2S$ level, $\delta E_p = e^2 r_p^2/(12a_0^3)$. Note that for the chosen quark charge density in our model, $r_o = r_p$. Thus the first term in Eq. (39) agrees with the accepted proton size term.

The second term in Eq. (39) is caused by polarization of the quarks by the electron. This polarization effect also causes an energy shift in the $2P$ energy level.[3] The difference between the energy shifts of the $2S$ and $2P$ levels might explain the proton radius puzzle.[4]

ACKNOWLEDGMENTS

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References

[1] http://www.electronformfactor.com/Extended Quark Effect On Hydrogen Spectrum(1S).

[2] http://www.fuw.edu.pl/ krp/papers/pohl.pdf

[3] http://www.electronformfactor.com/Extended Quark Effect On Hydrogen Spectrum(2P).

[4] http://www.electronformfactor.com/Extended Quark Effect On Muonic Hydrogen Spectrum.