EXTENDED QUARK EFFECT ON MUONIC HYDROGEN SPECTRUM

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1 A QUARK MODEL OF THE PROTON

The purpose of this paper is to test a theory of how the quarks in the proton might affect the H-atom spectra. There are three quarks making up the proton. There are two positively charged quarks, which are called up quarks, each one having a charge $q_+ = +2e/3$ where $-e$ is the electron charge. There is one negatively charged quark, which is called a down quark, and has a charge $q_- = -e/3$.

The quark charge is assumed to be uniformly distributed on a spherical shell of radius r_o. Thus the charge probability density of each quark is assumed to be $q_\pm \delta(y - r_o)/(4\pi r_0^2)$ where y is the distance from the quark center to a spherical shell of radius r_o and $\delta()$ is the Dirac delta function. Take the proton to be at the origin, and the proton radius to also be r_o. When only the proton is present, take the three quark centers to also be at the origin.

2 QUARK MODEL AND THE HYDROGEN ATOM

For a proton in the hydrogen atom, it will be assumed that the electron attracts the positively charged quarks, and repels the negatively charged quark. The positive quarks will be assumed to be displaced from the origin on the line from the origin to the electron. Label the displacement of the up quarks from the origin δ_+. The negative quark is assumed to be on the extension of the line from the electron through the origin. Label that displacement from the origin δ_-. In the papers cited below, it is argued that $\delta_- \approx \delta_+$.

The energy shift of the $2P$ level due to quark displacement is $-5e^2\delta_+/(36a_0^2)$ where a_0 is the Bohr radius.[1] This is in agreement with the results of a simpler quark model.[3,5]

The energy shift of the $2P$ level due to proton size is negligible in agreement with theory.[1] Thus the energy shift of the $2P$ level due to quark displacement and proton size is $\delta E_e(2P) = -5e^2\delta_+/(36a_0^2)$.

The energy shift of the $2S$ level due to quark displacement is $-5e^2\delta_+/(12a_0^2)$.[2]. This is in agreement with the results of the simpler quark model.[4,5] The energy shift of the $2S$ level due to proton size is $e^2r_0^2/(12a_0^3)$ in agreement with established theory.[9] Thus the total energy shift of the $2S$ level due to quark displacement and proton size is $\delta E_e(2S) = +e^2r_0^2/(12a_0^3) - 5e^2\delta_+/(12a_0^2)$.

The difference in the energy shifts is $\delta E_e = \delta E_e(2P) - \delta E_e(2S) = +e^2r_0^2/(12a_0^3) + 5e^2\delta_+/(18a_0^2)$. It will be assumed that $\delta_+/(a_0)^2$ is so small that it has negligible affect on the hydrogen atom spectrum. So the experimentally determined value $r_p = 0.8768\,\mathrm{fm}$ will be accepted as the root mean square proton radius.[6]

3 LAMB SHIFT IN MUONIC HYDROGEN [7,8,9]

The difference between the $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ energy levels of muonic hydrogen has been experimentally measured to be $\delta E_\mu = 206.2949\,\mathrm{meV}$. The theoretical difference is $\delta E_\mu = 209.9779\,\mathrm{meV}\text{-}5.2262\,r_p^2 + 0.0347\,r_p^3$ where r_p is the root mean square proton radius in fm. The r_p^2 term has been discussed in the previous sections of this paper. Equate the measured ΔE_μ to the theoretical ΔE_μ , and find $r_p = 0.84184\,\mathrm{fm}$.

4 QUARK MODEL AND MUONIC HYDROGEN

Apply the polarization of quarks described in sections (1) and (2) to muonic hydrogen. The energy shift ΔE_μ takes the same form as ΔE_e, but a_0 is replaced by the muon Bohr radius $a_{0\mu}$. The mass of the muon is approximately 200 times the mass of the electron, so $a_{0\mu}$ will be about 200 times smaller than a_0. Thus the muon will be closer to the quarks in muonic hydrogen than the electron in hydrogrn, and as a result, δ_+ will be larger in muonic hydrogen than in hydrogen. In addition, $1/a_{0\mu}^2 \approx 200^2/a_0^2$, so ΔE_μ is much larger than ΔE_e.

Refer back to section (3). Equate the measured energy difference between the two levels to the theoretical energy difference as in section (3), but add ΔE_μ to the theoretical energy difference. Thus

$$206.2949\,\mathrm{meV} = 209.9779\,\mathrm{meV} - 5.2262\,r_p^2 + 0.0347\,r_p^3 + 5e^2\delta_+/(18a_{0\mu}^2). \quad (1)$$

Substitute the accepted value $r_p = 0.8768\,\mathrm{fm}$ into the above equation, and solve for δ_+. The result is $\delta_+ \approx 5.45 \cdot 10^{-20}$ meters, which shows that a small polarization of the quarks can have a significant effect on the spectrum of muonic hydrogen.

Thus the polarization of the quarks caused by the presence of the muon provides a possible solution to the current proton radius puzzle. However, the model is too simple to be conclusive. It is hoped that a more realistic quark model will confirm the conclusions of this paper.

ACKNOWLEDGMENTS

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References

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